THEOREM 1 Properties of Limits

If L, M, c, and k are real numbers and

$$\lim_{x\to c} f(x) = L \quad \text{and} \quad \lim_{x\to c} g(x) = M, \text{ then}$$

1. *Sum Rule:* $\lim_{x\to c} (f(x) + g(x)) = L + M$

   The limit of the sum of two functions is the sum of their limits.

2. *Difference Rule:* $\lim_{x\to c} (f(x) - g(x)) = L - M$

   The limit of the difference of two functions is the difference of their limits.

3. *Product Rule:* $\lim_{x\to c} (f(x) \cdot g(x)) = L \cdot M$

   The limit of a product of two functions is the product of their limits.

4. *Constant Multiple Rule:* $\lim_{x\to c} (k \cdot f(x)) = k \cdot L$

   The limit of a constant times a function is the constant times the limit of the function.

5. *Quotient Rule:* $\lim_{x\to c} \dfrac{f(x)}{g(x)} = \dfrac{L}{M}, \; M \neq 0$

   The limit of a quotient of two functions is the quotient of their limits, provided the limit of the denominator is not zero.

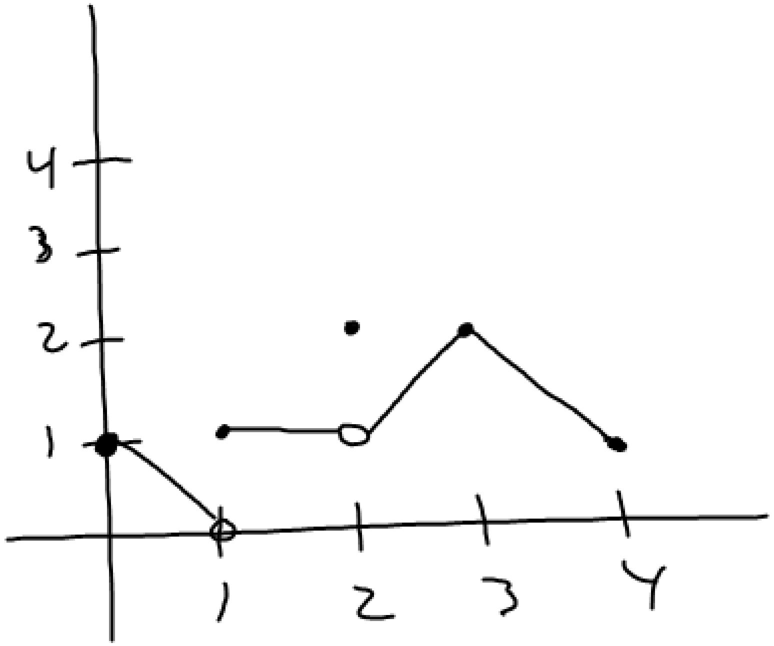
continued

If $\lim_{x\to c} f(x) = -\frac{1}{2}$ and $\lim_{x\to c} g(x) = 2/3$ Find $\lim_{x\to c}\left[f(x) - g(x)\right]$

$$= \cancel{-\frac{1}{6}},\ -\frac{7}{6}$$

$$-\frac{1}{2} - \frac{2}{3}$$

$$-\frac{3}{6} - \frac{4}{6} = -\frac{7}{6}$$



Find

$\lim_{x \to 0^+} f(x) = 1$

$\lim_{x \to 1^-} f(x) = 0$

$\lim_{x \to 2} f(x) = 1$

$\lim_{x \to 1} f(x) = \emptyset$

$\lim_{x \to 4^-} f(x) = 1$

① Use definition of limit to find the rate of change of $3x^2 - 2x$.

② Find $\lim_{\Delta x \to 0} \frac{\sqrt{(x+\Delta x)+2} - \sqrt{x+2}}{\Delta x}$

$f(x) = \sqrt{x+2}$

$f'(x) = \frac{1}{2\sqrt{x+2}}$

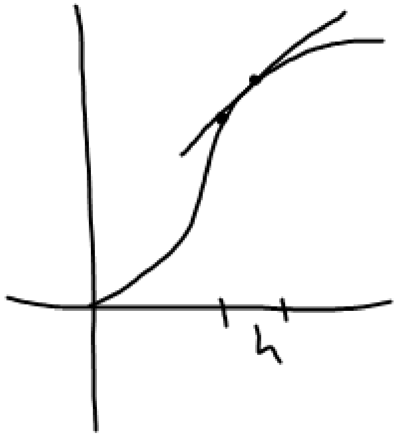
③ $\lim_{x \to 0} \frac{\tan x}{x} \longrightarrow \frac{f'(0)}{g(0)} = \frac{\sec^2(0)}{1} = \frac{1}{1} = \boxed{1}$

④ $\lim_{x \to 0} \frac{x}{\sqrt{9+x}-3} \longrightarrow \frac{f'(0)}{g'(0)} = \frac{1}{\frac{1}{2\sqrt{9+x}}} \Rightarrow 2\sqrt{9+0} = \boxed{6}$

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \Rightarrow \frac{3(x+h)^2 - 2(x+h) - (3x^2 - 2x)}{h}$$

$$\frac{\cancel{3x^2} + 6x\cancel{h} + 3h^{\cancel{2}} \cancel{- 2x} - 2\cancel{h} \cancel{- 3x^2} \cancel{+ 2x}}{\cancel{h}}$$

$$\lim_{h \to 0} 6x + 3h - 2 = \boxed{6x - 2}$$



$\boxed{2.2}$ Asymptotes & End Behavior

① Find the model for the end behavior

ⓐ $\dfrac{2x^5+x^4-x^2+1}{3x^2-5x+7}$

$\frac{2}{3}x^3$

ⓑ $\dfrac{2x^3-x^2+x-1}{5x^3+x^2+x-5}$

$= \frac{2}{5}$

② Find all vertical asymptotes without graphing it

$f(x) = \dfrac{2x-2}{\cancel{(x-1)}(x^2+x-1)} = 2\cancel{(x-1)}$ (Zeros of denominator* if Simplified)

Zeros $\cancel{1}$, $\frac{-1\pm\sqrt{5}}{2}$

removable discontinuity

Continuity

$\lim_{x \to c} f(x)$ exists

$f(c)$ exists

$f(c) = \lim_{x \to c} f(x)$

for all points in the domain

tan x is a continuous function

but there are intervals where

tan x is not continuous

Types of discontinuity

- jump
- oscillating
- removable
- infinite

THEOREM 6 Properties of Continuous Functions

If the functions f and g are continuous at $x = c$, then the following combinations are continuous at $x = c$.

1. *Sums:* $f + g$
2. *Differences:* $f - g$
3. *Products:* $f \cdot g$
4. *Constant multiples:* $k \cdot f$, for any number k
5. *Quotients:* f/g, provided $g(c) \neq 0$

THEOREM 7 Composite of Continuous Functions

If f is continuous at c and g is continuous at $f(c)$, then the composite $g \circ f$ is continuous at c.

4. If f is continuous for $a \le x \le b$ and differentiable for $a < x < b$, which of the following could be false?

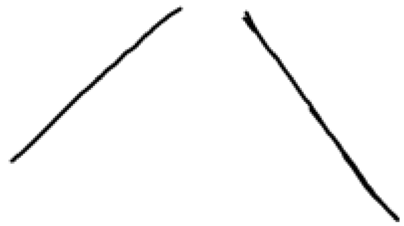
(A) $f'(c) = \dfrac{f(b) - f(a)}{b - a}$ for some c such that $a < c < b$.

(B) $f'(c) = 0$ for some c such that $a < c < b$.

(C) f has a minimum value on $a \le x \le b$.

(D) f has a maximum value on $a \le x \le b$.

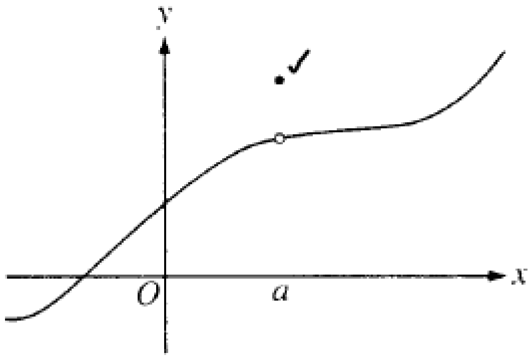
(E) $\int_a^b f(x)dx$ exists.



81. Let f be the function given by $f(x) = |x|$. Which of the following statements about f are true?

I. f is continuous at $x = 0$.
~~II.~~ f is differentiable at $x = 0$.
III. f has an absolute minimum at $x = 0$.

(A) I only ~~(B)~~ II only (C) III only (D) I and III only ~~(E)~~ II and III only



$f(a)$ exist ✓

$\lim_{x \to a} f(x)$ exist ✓

$f(a) = \lim_{x \to a} f(x)$ X

76. The graph of a function f is shown above. Which of the following statements about f is false?

(A) f is continuous at $x = a$.

(B) f has a relative maximum at $x = a$.

(C) $x = a$ is in the domain of f.

(D) $\lim_{x \to a^+} f(x)$ is equal to $\lim_{x \to a^-} f(x)$.

(E) $\lim_{x \to a} f(x)$ exists.